NASOQ: Numerically Accurate Sparsity-Oriented QP Solver

Supplemental Material

1 Settings for solvers

In this section we explain settings used for each solver and also discuss how these settings provide a fair evaluation. Throughout this section, `eps` refers to the accuracy threshold ϵ which is either 10^{-3}, 10^{-6}, or 10^{-9}.

1.1 Gurobi

The following parameters are set to `eps` for Gurobi [1]:

```
model_.set(GRB_DoubleParam_OptimalityTol, eps);
model_.set(GRB_DoubleParam_FeasibilityTol, eps);
```

Gurobi supports using an absolute termination criteria and setting these parameters provides a fair comparison with other solvers.

1.2 MOSEK

The following parameters are set to `eps` in MOSEK.[2]

```
program->setParamDouble(MSKdparam_enum::MSK_DPAR_INTPNT_QO_TOL_DFEAS,eps);
program->setParamDouble(MSKdparam_enum::MSK_DPAR_INTPNT_QO_TOL_PFEAS,eps);
program->setParamDouble(MSKdparam_enum::MSK_DPAR_INTPNT_QO_TOL_MU_RED,eps);
program->setParamDouble(MSKdparam_enum::MSK_DPAR_INTPNT_QO_TOL_INFEAS,eps);
```

MOSEK does not support an absolute termination criteria thus in addition to the demonstrated configuration, i.e., using `eps` for all parameters, we tested MOSEK with three different settings to find the best configuration for a fair enough evaluation:

- setting all `eps` to 10^{-16}.
- setting `MSKdparam_enum::MSK_DPAR_INTPNT_QO_TOL_INFEAS` to 10^{-12} and the remaining three parameters to `eps`.

[1] The specification of MOSEK termination criteria can be obtained from: `https://www.gurobi.com/documentation/8.1/refman/parameters.html`

[2] The specification of MOSEK's termination criteria can be obtained from: `https://docs.mosek.com/9.0/toolbox/param-groups.html#doc-termination-param-pargroup`

- setting all parameters to their default values for all requested accuracy thresholds, i.e., `eps`.

Our experiments show that using `eps` for all parameters provides an overall better failure rate and performance.

In addition to the above, we also realized using `eps` for parameter `MSKdparam_enum::MSK_DPAR_INTPNT_QO_TOL_REL_GAP` increases MOSEK's failure rate, so we did not configure this parameter and used its default value instead.

Parameter `MSKdparam_enum::MSK_DPAR_INTPNT_QO_TOL_NEAR_REL` shows when the MOSEK computed solution is optimal. This does not affect the number of iterations in MOSEK. We set it to `1.0` and use unified routines, which is used across solvers, to measure optimality for fair comparison.

1.3 OSQP

OSQP setting used in our experiments are: [3]

```
settings->linsys_solver = MKL_PARDISO_SOLVER;
settings->eps_abs=eps;
settings->eps_rel=eps;
settings->max_iter=40000000;
settings->eps_prim_inf = eps;
settings->eps_dual_inf = eps;
settings->polish = is_polish; // is set to 1 for OSQP-polished.
settings->verbose = 0;
settings->time_limit = 2000.0;
```

OSQP's default termination criteria is a relative measure, thus, we modify the OSQP code to support an absolute termination criteria for fair evaluation. Line 725 and 737 of `src/auxil.c` are changed and instead of calling `compute_pri_tol(work, eps_abs, eps_rel)` and `compute_dua_tol(work, eps_abs, eps_rel)` an absolute threshold is used with `eps_prim = eps_abs` and `eps_dual = eps_abs`. Functions `compute_pri_tol(work, eps_abs, eps_rel)` and `compute_dua_tol(work, eps_abs, eps_rel)` compute a relative threshold for primal and dual variables respectively. The modified code is provided with this document.

1.4 QL

For QL, we set the input `eps` parameter to the accuracy threshold `eps`.

2 Application-based breakdown

In this section, we show the failure rate and speedup of NASOQ compared to other solvers. To show the speedup, we use a geometric mean (GM). Speedup numbers are normalized to the geometric mean of NASOQ-Tuned, a larger the speedup value corresponds to a slower solver compared to NASOQ-Tuned.

[3] The osqp settings are available via this link, `https://osqp.org/docs/interfaces/solver_settings.html`.

Tables 1-4 show the results for four classes of QP problems in our QP repository. The total number of QP problems is 1513 which include 1308 contact simulation (Table 2), 53 Maros-Mészáros (Table 3), 120 model predictive control(Table 4), and 32 object deformation and model reconstruction (Table 5) problems.

	$\epsilon = 10^{-3}$		$\epsilon = 10^{-6}$		$\epsilon = 10^{-9}$	
	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)
Gurobi	1.1	3.4	3.9	6.8	14.3	27.5
MOSEK	13.5	60.7	92.3	>3000	97.3	>3000
NASOQ-Fixed	0.4	1.2	1	1.7	2.3	1.4
NASOQ-Tuned	0	1	0	1	1.5	1
OSQP	2.2	3.3	3	3.9	14.4	26.3
OSQP-polished	0.9	1.7	2	2.8	14	24.8

Table 1: All problems.

	$\epsilon = 10^{-3}$		$\epsilon = 10^{-6}$		$\epsilon = 10^{-9}$	
	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)
Gurobi	0.6	3.5	3.5	8.2	10.1	21.8
MOSEK	13.5	87.2	94.3	>3000	98.1	>3000
NASOQ-Fixed	0	1	0.2	1.2	0.8	1.1
NASOQ-Tuned	0	1	0	1	0.6	1
OSQP	1.2	2.5	1.1	2.1	13.8	42.6
OSQP-polished	0.6	1.8	0.6	1.7	13.8	42.9

Table 2: Contact simulation problems.

	$\epsilon = 10^{-3}$		$\epsilon = 10^{-6}$		$\epsilon = 10^{-9}$	
	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)
Gurobi	13.2	9.8	15.1	8.9	58.5	2166.7
MOSEK	34	136.8	79.2	>3000	88.7	>3000
NASOQ-Fixed	9.4	3.2	24.5	28.8	35.8	25.3
NASOQ-Tuned	0	1	0	1	15.1	1
OSQP	24.5	31.9	45.3	989	47.1	182.7
OSQP-polished	9.4	1.9	35.8	183	40	45.2

Table 3: Maros-Mészáros problems.

	$\epsilon = 10^{-3}$		$\epsilon = 10^{-6}$		$\epsilon = 10^{-9}$	
	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)
Gurobi	0	0.4	0.8	34.3	19.2	>3000
MOSEK	0.8	0.4	86.7	>3000	100	>3000
NASOQ-Fixed	0	1	0	1	0	1
NASOQ-Tuned	0	1	0	1	0	1
OSQP	3.3	12.7	2.5	119.3	2.5	123
OSQP-polished	0	61.1	0	3.4	0	4

Table 4: Model Predictive Control (MPC) problems.

	$\epsilon = 10^{-3}$		$\epsilon = 10^{-6}$		$\epsilon = 10^{-9}$	
	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)	Failue rate (%)	Speedup (GM)
Gurobi	6.3	10.4	12.5	24	90.7	>3000
MOSEK	28.1	231.3	50	>3000	68.8	>3000
NASOQ-Fixed	3.1	1.7	0	1	18.8	1
NASOQ-Tuned	0	1	0	1	18.8	1
OSQP	3.1	4.2	12.5	22.8	28.1	6.2
OSQP-polished	0	2.2	12.5	23.3	34.4	19.6

Table 5: Model reconstruction and object deformation problems.